

**Jozef MELCER<sup>1</sup>****INFLUENCE OF DAMPING ON FRF OF VEHICLE COMPUTING MODEL****Abstract**

Frequency Response Function (FRF) characterizes the response of dynamic system in frequency domain. Its shape is dependent on the property of analyzed system, especially on damping. The damping neglecting during the calculation of frequency response causes that the Power Response Factors have the character of discrete spectral lines at points of natural frequencies of the vehicle computing model.

**Keywords**

Vehicle computing model, Frequency Response Function, power response factor, influence of damping, numerical solution.

**1 INTRODUCTION**

The roads are the typical transport structures subjected to dynamic effect of moving transport means. Road surface unevenness represents the main source of kinematical excitation of vehicle so it represents the main source of vehicle dynamic effect on pavements. The problem can be analyzed by numerical or by experimental way. But the best way is the mutual combination of both mentioned advances. Numerical analysis can be realized in time or in frequency domain. The complex approach requires the analysis in time and also in frequency domain. The vehicle computing models can be created on various levels. Generally the 3D – full, 2D – half and 1D – quarter computing model of vehicle can be created. The basic information about FRF, 2D vehicle computing model, its equations of motions and about possibilities of transfer from time to frequency domain are introduced in reference [1]. Also the results on numerical solution of FRF were presented there. The mutual comparison of FRF calculated for 3D, 2D and 1D computing models of vehicle were presented in [2]. It was illustrated there that when the vehicle computing models are mutually equivalent the power response factors are practically identical. In the submitted paper the influence of damping on the shape and functional values of FRF is analyzed.

**2 VEHICLE COMPUTING MODEL**

In this paper the 2D – half computing model of vehicle is analyzed, Fig. 1. The computing model has 5 mass degrees of freedom. The natural frequencies of this computing model corresponding to fully tracked vehicle Tatra 815 are as follows:  $\{\mathbf{f}\} = \{f_{(1)}; f_{(2)}; f_{(3)}; f_{(4)}; f_{(5)}\} = \{1,13; 1,45; 8,89; 10,91; 11,71\}$  [Hz]. These are the natural frequencies of non-damped vibration [3].

**3 FREQUENCY RESPONSE FUNCTIONS**

Frequency response of linear system (Frequency Response Function  $FRF(p)$  where  $p = i \cdot \omega$  is a complex number, Fig. 2) is defined as ratio of steady state response and harmonic excitation. When the excitation  $h(t)$  is harmonic with unit amplitude

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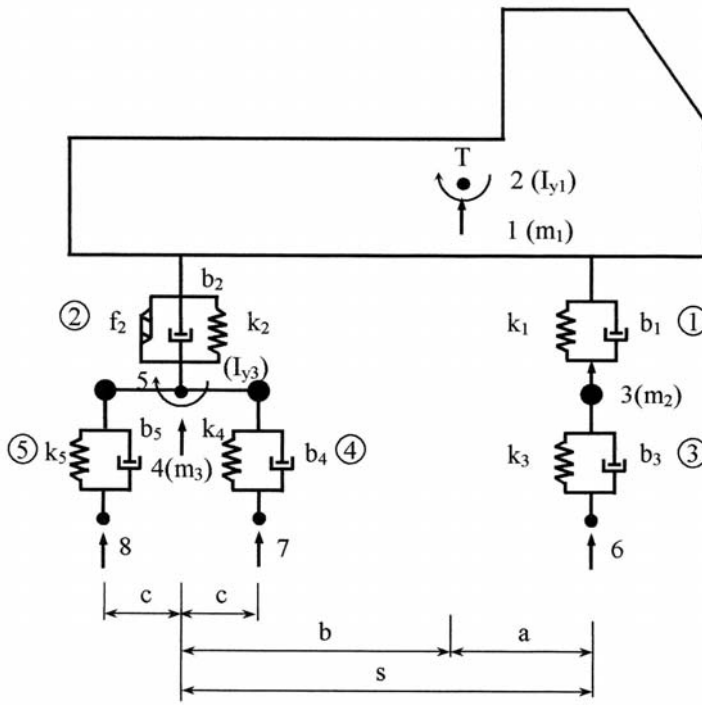


Fig.1: 2D – half computing model of vehicle

$$h(t) = h \cdot f(t) = 1 \cdot e^{i \cdot \omega \cdot t} \quad (1)$$

then we can write

$$FRF(p) = FRF(i \cdot \omega) = r_{st}(t) / (h \cdot e^{i \cdot \omega \cdot t}) = r_{st}(t) / (1 \cdot e^{i \cdot \omega \cdot t}) = r_{st}(t) \cdot e^{-i \cdot \omega \cdot t}. \quad (2)$$

The Frequency Response Function, as the complex function, can be shown as vector sum of real and imaginary part.

$$FRF(p) = \text{Re}[FRF(p)] + i \cdot \text{Im}[FRF(p)], \quad (3)$$

or

$$FRF(p) = |FRF(p)| \cdot e^{i \cdot \varphi}, \quad (4)$$

where  $|FRF(p)|$  is absolute value, or amplitude of Frequency Response Function. It is valid that

$$|FRF(p)| = \sqrt{\text{Re}^2[FRF(p)] + \text{Im}^2[FRF(p)]}. \quad (5)$$

The second power of absolute value of Frequency Response Function  $|FRF(p)|^2$  is called as Power Response Factor (PRF).

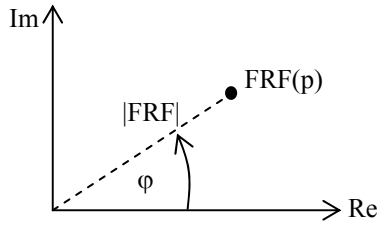


Fig. 2: Graphical interpretation of FRF

#### 4 RESULTS OF NUMERICAL ANALYSIS

Numerical solution of Frequency Response Functions was carried out for 2D – half computing model of vehicle. All FRF are referred to kinematical excitation due to road unevenness  $h_3$  under front wheel of vehicle. The speed of vehicle motion is equal  $v = 10$  m/s. All computations were carried out in the environment of the program system MATLAB [4]. The Power Response Factors of dynamic component of tire force under each wheel of vehicle are monitored for the following 3 variants:

1. The damping is assumed in full values for all connecting members of vehicle computing model,  $b_1 \div b_5 \neq 0$ .
2. The damping in tires is neglected,  $b_3 \div b_5 = 0$ .
3. The damping is neglected for all connecting members of vehicle computing model,  $b_1 \div b_5 = 0$ .

The values of damping coefficients for 2D vehicle computing model are as follows:

$$b_1 = 9614 \text{ kg/s}, \quad b_2 = 130098,5 \text{ kg/s}, \quad b_3 = 1373 \text{ kg/s}, \quad b_4 = 2747 \text{ kg/s}, \quad b_5 = 2747 \text{ kg/s}.$$

In Fig. 3 the PRF of dynamic component of tire force  $F_{dyn,3}$  for 3 various variants of damping are introduces:

- with damping	$f = 9.20 \text{ Hz},$	$ F_{dyn,3}/h_3 ^2 = 8.693 \cdot 10^{12} \text{ N}^2/\text{m}^2,$
- without damping in tires	$f = 9.12 \text{ Hz},$	$ F_{dyn,3}/h_3 ^2 = 11.115 \cdot 10^{12} \text{ N}^2/\text{m}^2,$
- without damping	$f = 8.89 \text{ Hz},$	$ F_{dyn,3}/h_3 ^2 = 2.201 \cdot 10^{18} \text{ N}^2/\text{m}^2.$

In Fig. 4 the PRF of dynamic component of tire force  $F_{dyn,4}$  for 3 various variants of damping are introduces:

- with damping	$f = 10.94 \text{ Hz},$	$ F_{dyn,4}/h_3 ^2 = 10.657 \cdot 10^{14} \text{ N}^2/\text{m}^2,$
- without damping in tires	$f = 10.91 \text{ Hz},$	$ F_{dyn,4}/h_3 ^2 = 8.471 \cdot 10^{18} \text{ N}^2/\text{m}^2,$
- without damping	$f = 10.91 \text{ Hz},$	$ F_{dyn,4}/h_3 ^2 = 8.469 \cdot 10^{18} \text{ N}^2/\text{m}^2.$

In Fig. 5 the PRF of dynamic component of tire force  $F_{dyn,5}$  for 3 various variants of damping are introduces:

- with damping	$f = 10.93 \text{ Hz},$	$ F_{dyn,5}/h_3 ^2 = 11.235 \cdot 10^{14} \text{ N}^2/\text{m}^2,$
- without damping in tires	$f = 10.91 \text{ Hz},$	$ F_{dyn,5}/h_3 ^2 = 8.467 \cdot 10^{18} \text{ N}^2/\text{m}^2,$
- without damping	$f = 10.91 \text{ Hz},$	$ F_{dyn,5}/h_3 ^2 = 8.469 \cdot 10^{18} \text{ N}^2/\text{m}^2.$

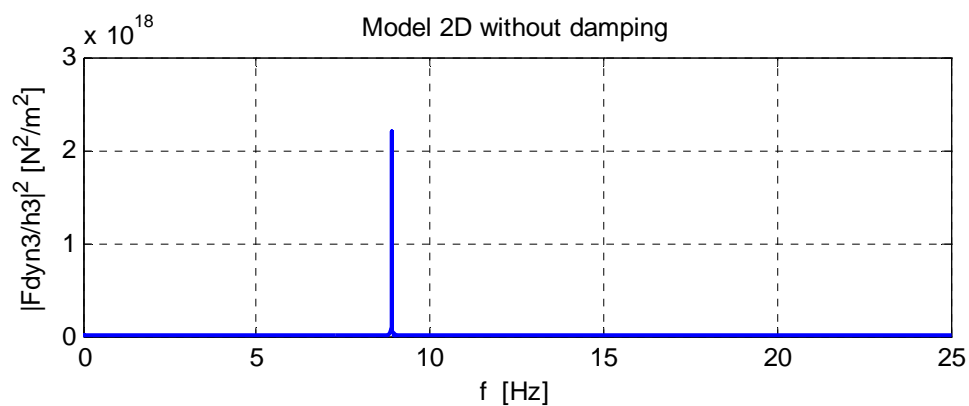
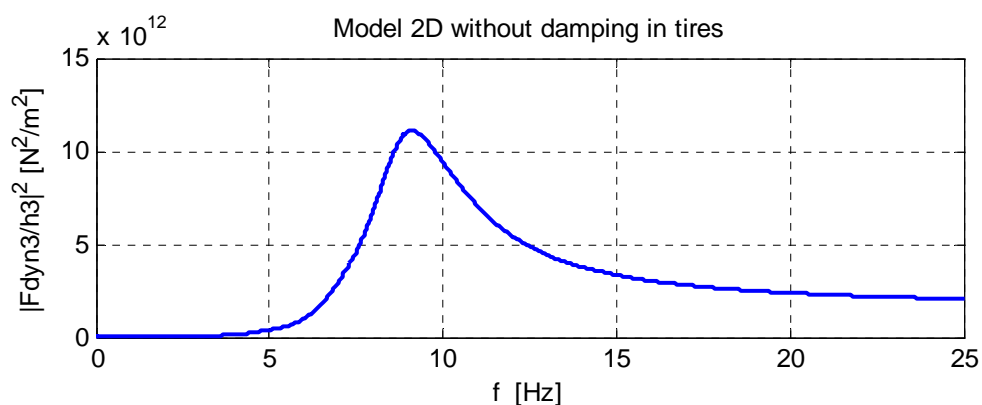
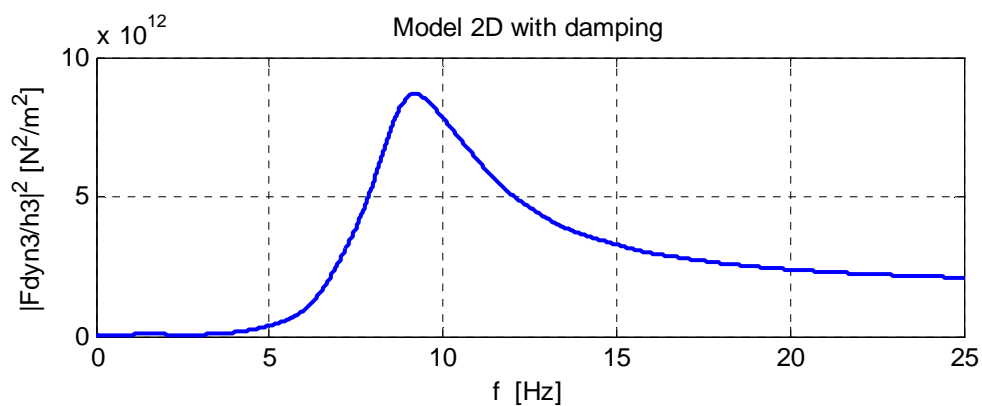


Fig. 3: PRF of dynamic component of tire force  $F_{dyn,3}$  under front axle for 3 variants of damping

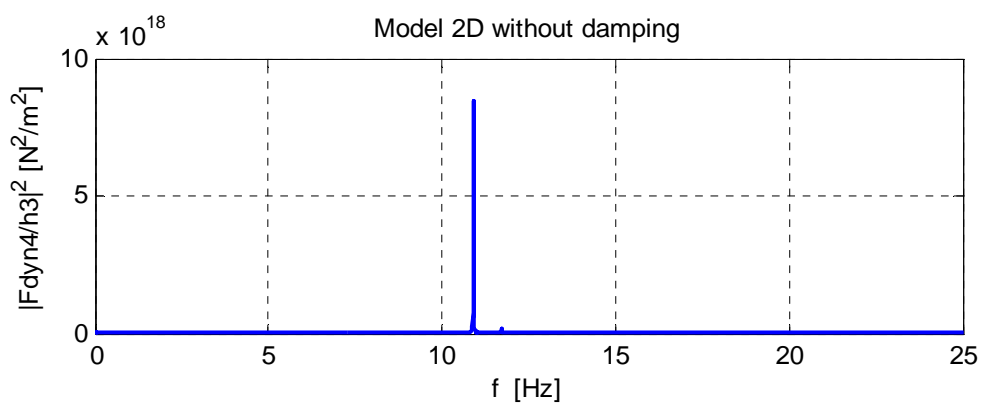
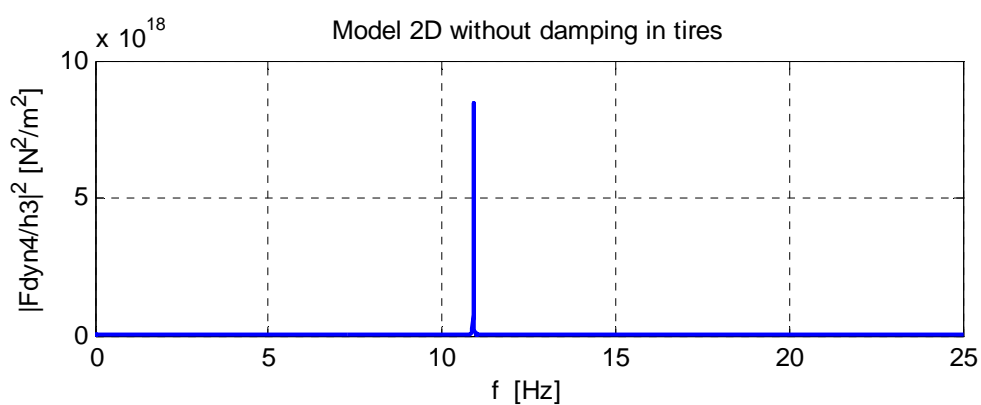
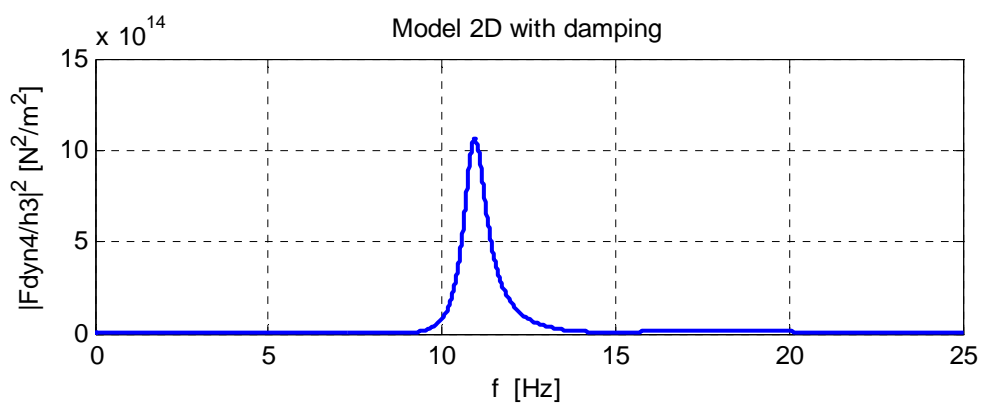


Fig. 4: PRF of dynamic component of tire force  $F_{dyn,4}$  under front wheel of rear axle for 3 variants of damping

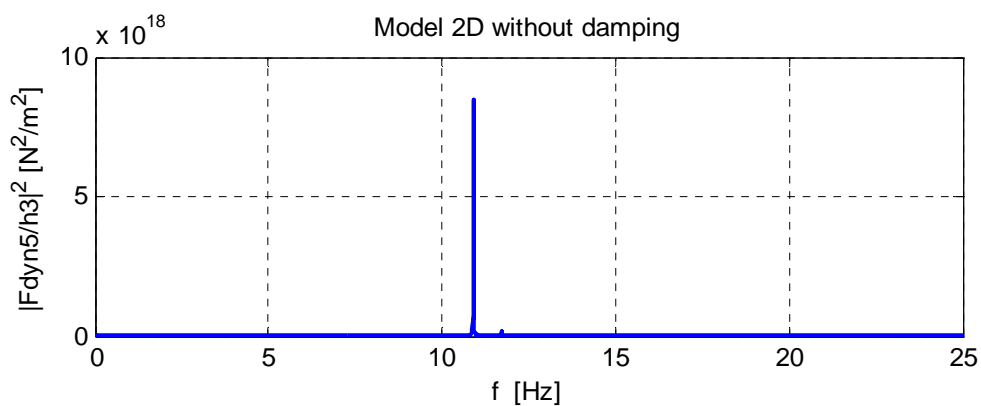
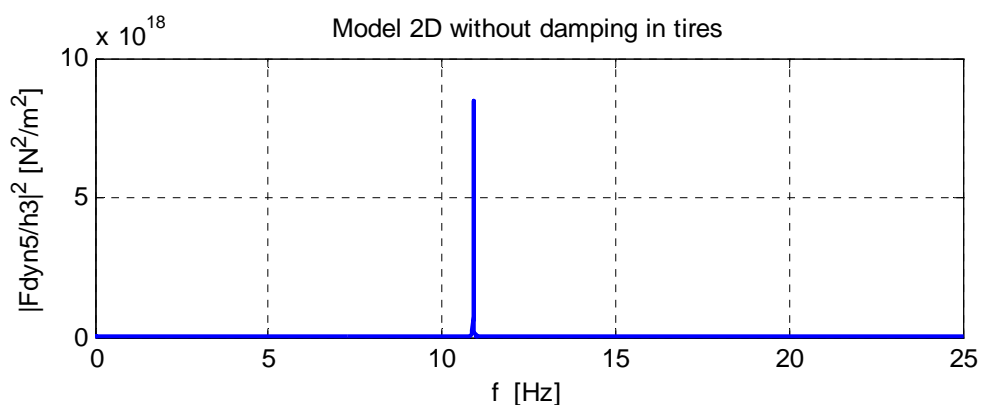
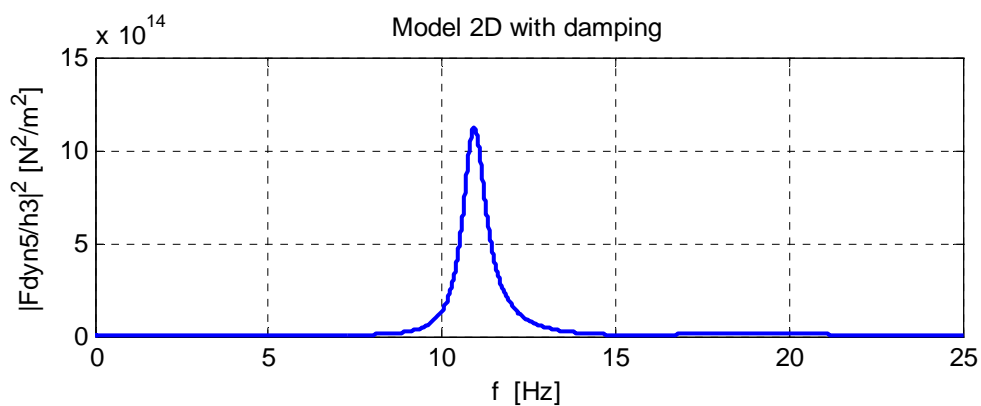


Fig. 5: PRF of dynamic component of tire force  $F_{dyn,5}$  under rear wheel of rear axle for 3 variants of damping

## 5 CONCLUSIONS

One of the monitored values within the solution of dynamic tasks in frequency domain is the Frequency Response Function. The Frequency Response Function describes the properties of analyzed dynamic system in dependence on the value of exciting frequency. Frequently the second powers of absolute values of FRF are considered. There are called Power Response Factors. Damping is one of the most important parameter describing the properties of a dynamical system. In the submitted paper the influence of damping on the shape and values of Power Response Factors of dynamic component of tire forces for 2D computing model of vehicle is analyzed. When the damping is neglected the Power Response Factors have the character of discrete spectral lines at points of natural frequencies of the vehicle computing model. The functional values of monitored values are increased from 4 to 6 places value of digit. On the opposite the damping causes expansion these characteristics to sides on the area of the adjacent frequencies. The functional values of monitored values are decreased.

## ACKNOWLEDGMENT

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